USING A RESIDUAL STRENGTH – LRDF METHODOLOGY TO PREDICT THE DURABILITY OF COMPOSITE MATERIALS USED IN CIVIL APPLICATIONS

John V. Bausano and John J. Lesko
Materials Response Group
120 Patton Hall
Department of Engineering Science and Mechanics
Virginia Tech
Blacksburg, VA 24060

Abstract

Design guidelines for fiber-reinforced polymers (FRP) that are used in the construction of civil applications must take into account the loss of stiffness and strength these materials undergo over time. American Association of State Highway Transportation Officials (AASHTO) load resistance factor design (LRFD) uses statistical methods to develop a strength reduction factor for a particular combination of loads. LRFD uses statistical representations of mechanical properties at a specific time coupled with a statistical representation of the loading situation to determine the appropriate resistance factors. LRFD alone is an inadequate way to determine the future reliability of a composite structure due to the lack of knowledge about the future properties of the composite structure. A new method that combines LRFD and a material properties evolution scheme is proposed to predict the future reliability of a FRP structure.

The Reifsnider residual strength model is used to track the remaining strength of the structure through time. The residual strength model is capable of taking into the account the variability of the loading conditions, mechanical and environmental. Inputs that are unique to this method are the statistical representations of a materials S-N curve and stiffness loss. The strength evolution model is conducted in a Monte Carlo style to determine a statistical representation of the remaining strength at some future time. Using this distribution of remaining strength as input, the LRFD methods can be used to determine the reliability and the resistance factors for the structure at a specific time in the future.

Introduction

FRP composite structures have gained a limited amount of acceptance in civil applications through experimental structural implementations. To further increase the demand for these structures new design standards and guidelines must be available for routine implementation of these innovative systems. Load resistance factor design (LRFD) is a probability based design method that is presently being adopted into regular design practices for conventional material structures. There is also a significant interest in developing a design standard for FRP composites that is consistent with the LRFD standard.

Separately various suppliers have developed design manuals of various levels (e.g. Hardcore Composites, Strongwell, Corp, Hughes Brothers). These material qualifications provide the user with direction in the component's correct use and allowable loads/stresses under various use conditions. Each clearly defines a material, dimensions and tolerances for the component and design equations or tables for various limit states in suitable implementations.

One element missing from these initial efforts is the realization that over time the strength and stiffness will change due to combined exposure to load and environment. A major concern when dealing with such changes in the context of an allowable for a design is the reality of potential failure mode changes and how to deal with such events. Although the LRFD approach provides a clear and logical statistical approach to design, it also relies on the existence of one well-defined failure mode and does not account for a change in failure mode.

Durability and LRFD Approach

In the LRFD approach the probability distribution of loads/stress (Loads, Q) is compared to the probability of failure strength of the material (Resistance, R), typically represented as illustrated in Figure 1(AASHTO 1998). The region of overlap *is related to the risk* associated with the situation designed. Selecting the form and size of the structure determines the desired overlap of the two distributions thus defining the stated allowable risk for a given loading condition.

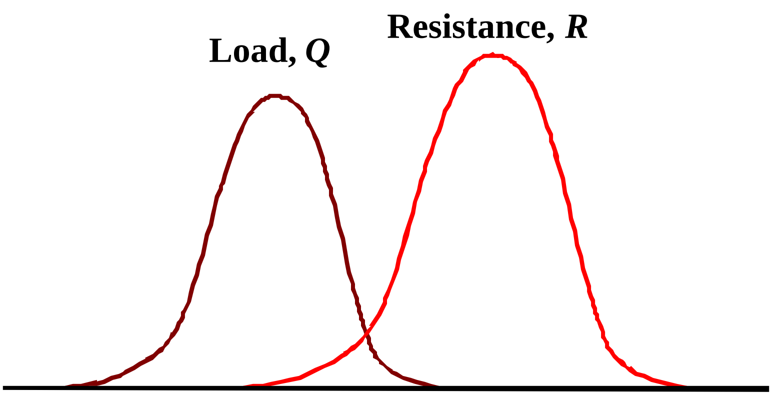

Figure 1: Weibull load and resistance distributions

The basis for these calculations when dealing with composite structures lies in Weibull statistics (Weibull 1951,Weibull 1949), where the cumulative probability distribution function describing the distribution of measured values is given by,

$$F(x) = 1 - e^{-\left(x/\beta\right)^{\alpha}} \tag{1}$$

where, α and β are the two parameters used to fit the data. The value of α (the shape parameter) determines the breath of the distribution while β (the location parameter) defines the value most closely representing the center of the distribution. Based on the concept of reliability, R(x) the probability of failure, F(x), is related to the reliability by,

$$R(x) = 1 - F(x) \tag{2}$$

The reliability of composite structures can change over time because of the strength and stiffness loss the systems exhibit. The loss of reliability defines new lower factors of safety associated with the systems. As these FRP structural shapes are new to the industry, definitive criteria for reasonable factors of safety based on durability are presently not available.

With this incomplete picture of how one should account for strength and stiffness reductions in the design of FRP structures we began the development of an LRFD based simulation approach. Using the commonly reported expression,

$$f R_n > \sum g_i Q_{ni} \tag{3}$$

which describes the basis of the LRFD philosophy, we attempt to simulate and track the reduction in the Resistance, R, in the presence of a particular load environment, Q, including both mechanical and environmental stressors. We hypothesize that in the presence of a particular set of factored loads/environments the resistance will change in one of two ways as well as their combination as illustrated in Figure 2. We can suggest that the probability density function (PDF) of the resistance will change the initial representation of the resistance by either 1) simply shifting to lower strength (A of Figure 2), 2) through the change in shape of the PDF (B of Figure 2) or 3) through a combination of A & B changes in the residual resistance.

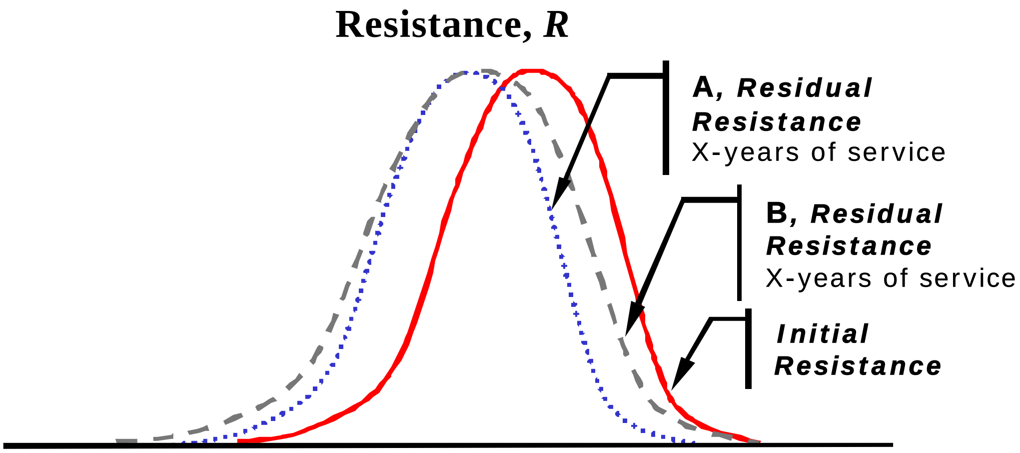

Figure 2: The hypothesized changes in the probability density function describing the statistical distribution in the resistance of a component or material.

Possessing such representations we can go through the normal procedure of computing the probability of failure, P_f, and the partial safety factor ϕ of equation 3. This is accomplished through the representation shown in Figure 3. When Q>R failure occurs, the probability that this limit state occurs is given by equation 4. Here the probability of failure is determined from the integral of the overlapping PDF for the loads and the cumulative distribution function (CDF) by,

$$P_f = \int PDF_Q \cdot CDF_R \tag{4}$$

The shaded area bounded by the CDF and the PDF is related to the probability of failure. The reduction factor of Equation (3) can then be computed from,

$$\phi = \left(\frac{\mu_R}{R_n}\right)\exp\left(-\alpha_R \beta V_R\right) \tag{5}$$

$$\beta = \frac{\ln\left(R_n / Q_n\right)}{\sqrt{V_R^{\,2} + V_Q^{\,2}}} \tag{6}$$

for lognormal representations of the resistance and loads. For the above equations, μ_R is the mean of the resistance, α_R is a “sensitivity coefficient” that depends on the PDF of the resistance curve (typically between 0.5 and 0.6), V_R and V_Q are the coefficients of variation for the resistance and load distributions respectively.

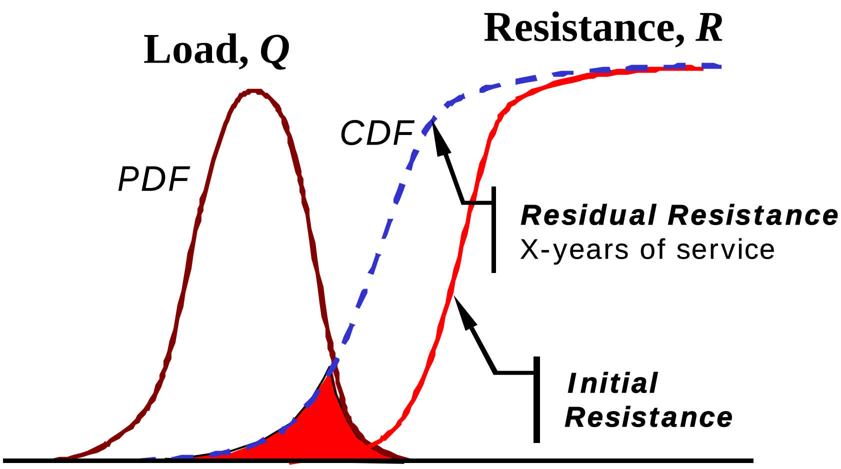

Figure 3: Representation of the probability of failure calculation for the LRFD approach including the influence of changes in the initial resistance due to service

Such an expression can be derived for Weibull statistical representations of load and resistance by the use of first order reliability methods (FORM). An equivalent method of describing the limit state failure probability is by defining a limit state function,

$$G(X_1, X_2, \ldots, X_m) = 0 \tag{7}$$

where X_m is a random load or resistance variable. By convention failure occurs when G(X)<0, for this case G = R-Q. FORM defines the reliability index as,

$$b = \frac{mG}{s\,G} \tag{8}$$

where µG and σG are estimates of the mean and standard deviation of the limit state function after it has been linearized around an appropriate point on the surface G(X)=0.

The question that remains unanswered is how to simulate these changes in residual resistance. The residual strength approach developed and advanced by Reifsnider & co-workers (Reifsnider et al. 1995, Reifsnider et al. 1996) is proposed to predict the residual strength.

$$Fr(n) = Fr(0) - \int_0^{\frac{n}{N}} (1 - Fa(n)) \cdot J \cdot \left(\frac{n}{N}\right)^{J-1} \cdot d\left(\frac{n}{N}\right) \tag{9}$$

where,

n = elapsed number of cycles
N = life in cycles at applied stress
Fa = normalized applied stress
Fr = normalized remaining strength
J = curve fit parameter

N, the cycles to failure at an applied stress level can be calculated using

$$N(Fa) = 10\wedge\left(\frac{Fa - A}{B}\right)^{\frac{1}{p}} \tag{10}$$

where,

A, B curve fit parameters from the materials S-N curve
p is assumed to be 1

This method of life prediction for polymer composites possesses the ability to combine loads and environments to develop a representation of the residual properties of the composites as a function of time and or cycles (service condition). The approach relies on the mechanistic representation of changes in constituent properties and the presence and development of damage as a result of the applied stresses and environments. Tracking residual strength provides a metric which is compared to the resulting stress level in the critical element (the ply or interface that controls the failure of the composite). When the remaining strength of the critical element equals the stress applied to it, failure occurs. In addition, residual stiffness is tracked in this analysis and can offer an estimate of deflection limit states and they are typically the controlling feature in an FRP structural design. Thus, the necessary tools are available to carry out a simulation that can be used to simulate and suggest appropriate ϕ factors for various applications and service environments. The proposed simulation is illustrated in Figure 4.

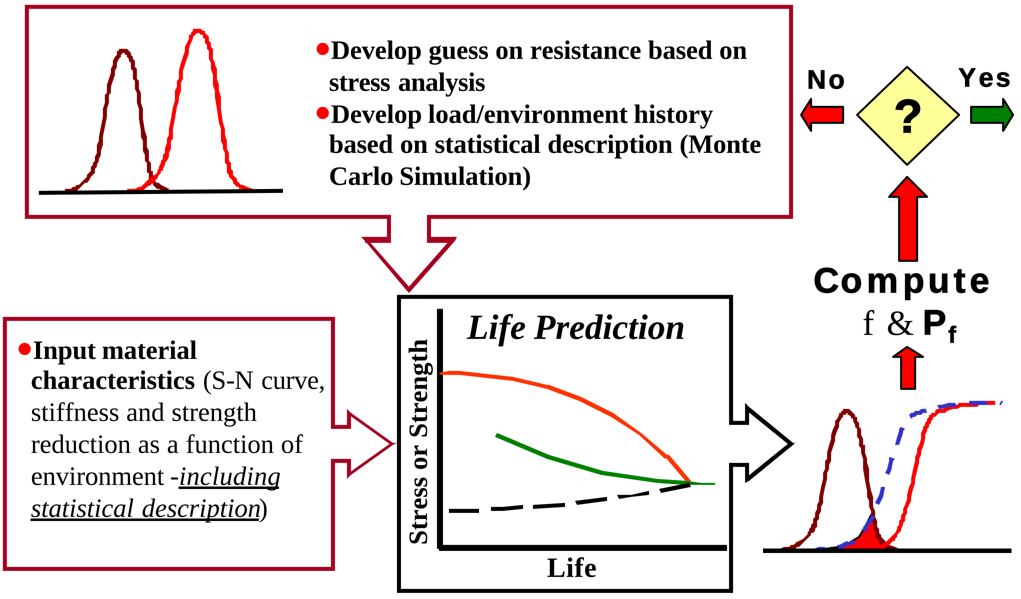

Figure 4: Proposed simulation for the assessment of acceptable f factors.

The simulation begins by securing a representation of the loads (including the hygro-thermal service environment). For purposes of representing this load and environment history in the model we employ a Monte Carlo simulation. Based on the estimated loads, a structural design is completed and the stress analysis allows for the assessment of the condition that controls the design. This stress analysis is used to feed the residual strength/stiffness life prediction model discussed above. Here the life prediction model must be supplied with representations of the functional forms of stiffness and strength loss as a function of exposure to individual degrading phenomena. Work continues in the area of understanding how the combination and synergism of environment and load. However, there are presently some reasonable approximations of how to treat some classical combinations of load and moisture and temperature for glass/polymer composites (McBagonluri et al. 2000, Phifer et al. 2000). Furthermore, our laboratory continues to examine simulations of realistic service environments under accelerated conditions to understand the influence of synergistic effects. These findings will be incorporated in futures efforts to assess durability and codes development. The simulation also requires that we include statistical representations of the phenomena and their combination. This is particularly true of fatigue and those environmental features that influence variability and shift in the phenomena.

Having defined these input, the simulation will be conducted and develop the CDF of the residual resistance shown in Figure 3 for a predefined period of service. Computing the probability of failure and the associated ϕ, we are now left to make a judgment call as to the acceptable level of risk. If the engineer or the community believes that the risk is reasonable for the period chosen, then we are satisfied with the material and its use in the design and select the computed ϕ for the design. If on the other hand the engineer finds the level of risk too high, then he/she is left with the ability to make a second guess on the design to change component sizes or material properties to further separate the resistance from the loads. Repeating the simulation the risk and ϕ are computed and checked again till the engineer arrives at an acceptable level of risk.

With the availability of such a tool the community could develop a series of ϕ's for various material specified FRP components under various exposure to various regions of the U.S., and traffic characteristics. (The life and therefore the ϕ will be dependent on the load and environmental service condition for the structure.) This could potentially provide an engineer with a table from which to select a ϕ factor appropriate for the design, removing the engineer from the need to run the simulation. Ideally this allows the bridge design community to meld the LRFD approach for conventional structures, which is gaining acceptance, to those that will be developed in the future that include FRP composites.

Preliminary Model Implimentation

Experimental Data Collection

A first implementation of the proposed simulation technique was performed on a pultruded E-glass / Dow Derakane Momentum 610-900 resin composite laminate. The laminate had a lay-up of [csm/0°/90°/csm/±45° /csm]s, where csm stands for continuous strand mat. The initial quasi-static stiffness and strength was determined experimentally for the material system along with the fatigue performance (S-N curve). The laminate stiffness loss was also tracked for each fatigue sample that was tested to construct the S-N curve.

The quasi-static properties were determined from 15 standard tensile tests, the large number of samples used allowed for the determination of the statistical variation in the initial strength and stiffness of the material. The results are listed in Table 1.

Table 1: Quasi-Static Laminate Properties

Weibull Parameter	Stiffness	Strength
α	17.7	4.9
β (PSI)	2.24 E 6	34.8 E 3

To determine the S-N curve 100 individual fatigue tests were performed at an R ratio of 0.1 (min applied stress / max applied stress). The tests were conducted at a wide range of stress ratios (%UTS), which is defined as the maximum applied stress divided by the initial strength. The resulting S-N curve is shown in Figure 5. Again with the large numbers of samples tested at each stress level Weibull statistics can be determined for the variability in cycles to failure at a specific stress level. These calculated Weibull parameters are presented in Figure 5 for multiple stress levels.

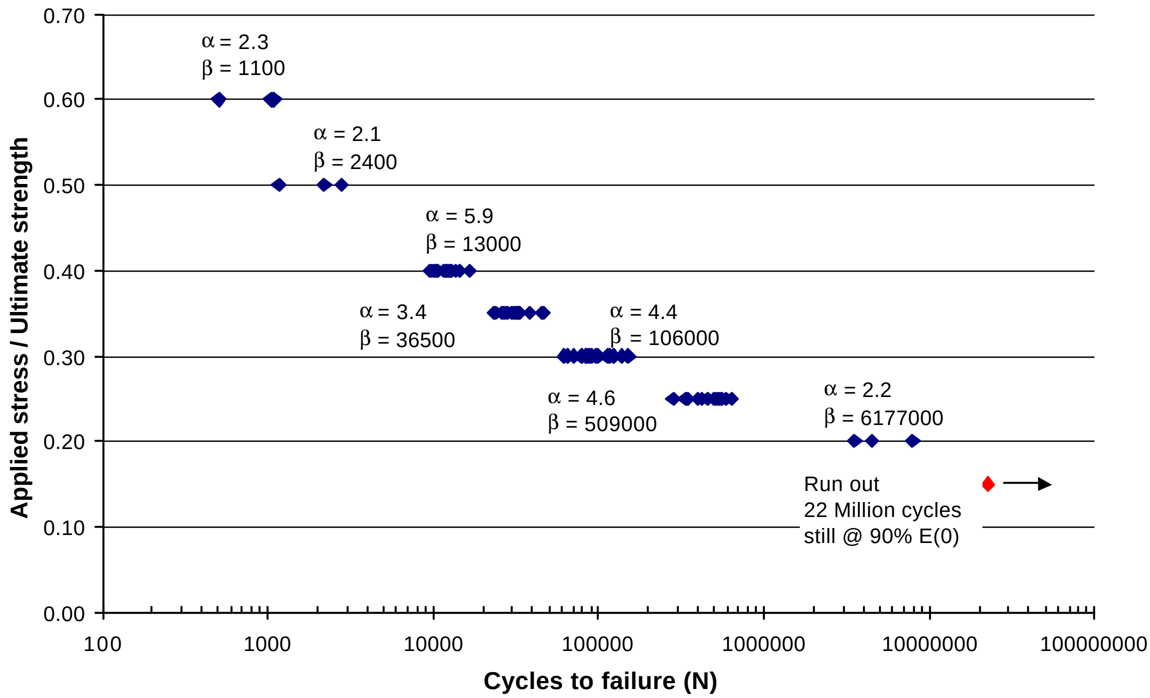

Figure 5: Laminate S-N curve

In a similar way Weibull parameters can be fit to the stiffness loss data. The rate at which samples lose stiffness is dependant upon the magnitude of the applied stress; the greater the applied stress the faster the stiffness loss and vice-versa. Because of this fact individual stiffness loss curves were grouped together by which stress level each was tested at. This allowed for the determination of the statistical variation in the modulus reduction at different % lifetimes of the sample set at each stress level. Figure 6 is a plot of the laminate stiffness ruction plots for the 35% UTS stress level, notice the normalization of the cycles to failure. As shown in Figure 6, at a specific time in normalized lifetime, there exists a definite distribution in stiffness loss. Weibull parameters were calculated to describe this behavior at 1, 5 10, 20, 50, 80, 90, 95 and 99 percent lifetime. An example of such a data set for the 35% stress level is depicted in graphically in Figure 7. Figure 6 and Figure 7 show the "classic" shape at which a laminate of this type losses stiffness. For the first 10% of life the stiffness loss occurs quickly. This is followed by a gradual linear loss in stiffness. The stiffness loss stays linear until right before failure where large losses in stiffness are observed. The data that was calculated for the 35% UTS level in Figure 7 was also calculated for UTS levels of 25 and 30%. The Weibull parameters were plotted over normalized cycles to failure, see Figure 8.

Modeling Results

Using gathered data the simulation was implemented to determine the distribution of the remaining strength of a simple tensile fatigue process. The simulated test was a fatigue loading at 35% UTS for 18750 cycles, 18750 cycles is two standard deviations (6465 cycles) lower than the mean cycles to failure at 35% UTS, Figure 5. The simulation was run for 500 trials, each time calculating the residual strength at the 18750 cycles. From the simulation the Weibull parameters for the variation in remaining strength were determined to be $\alpha = 5.8$ and $\beta = 23.9$ E 3 PSI or 69% of the original non-fatigued strength. The model also predicted that 23% of the samples would fail before they reached the 18750 cycle mark.

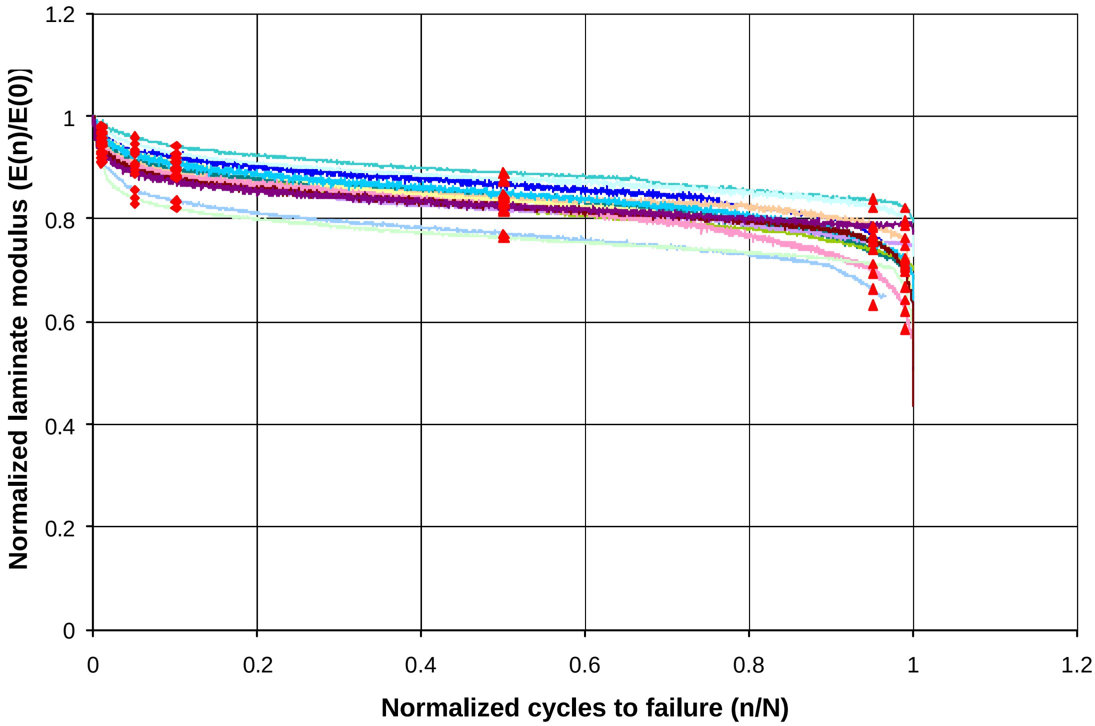

Figure 6: Normalized modulus reduction for 35% UTS

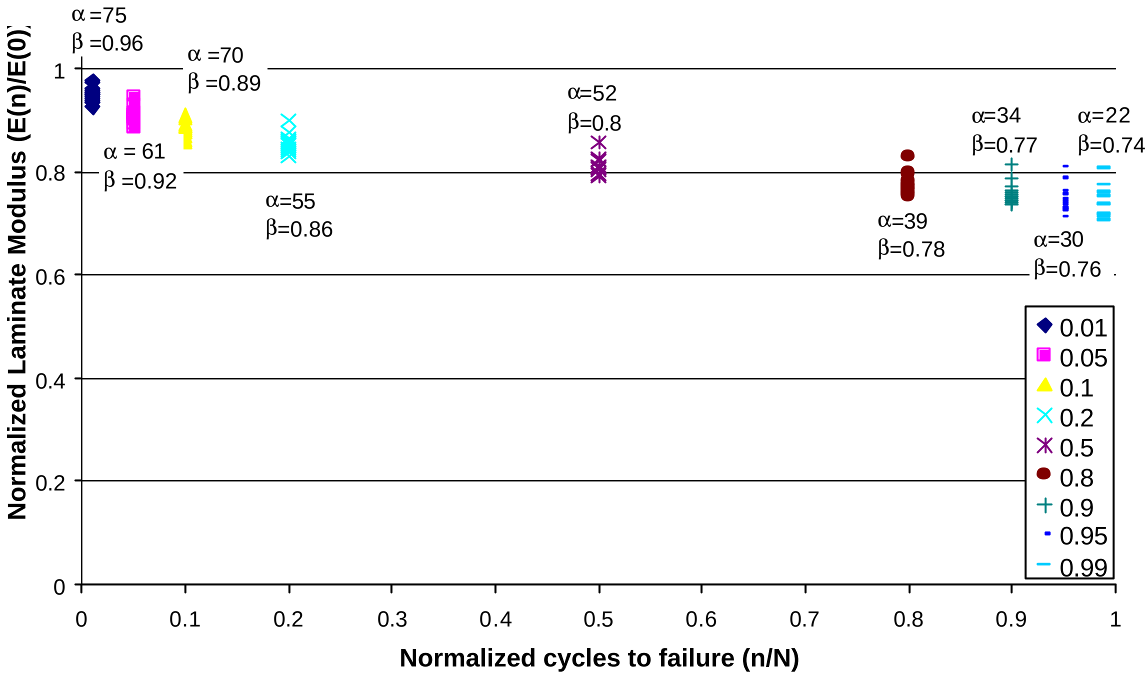

Figure 7: Normalized laminate modulus reduction for 35% UTS

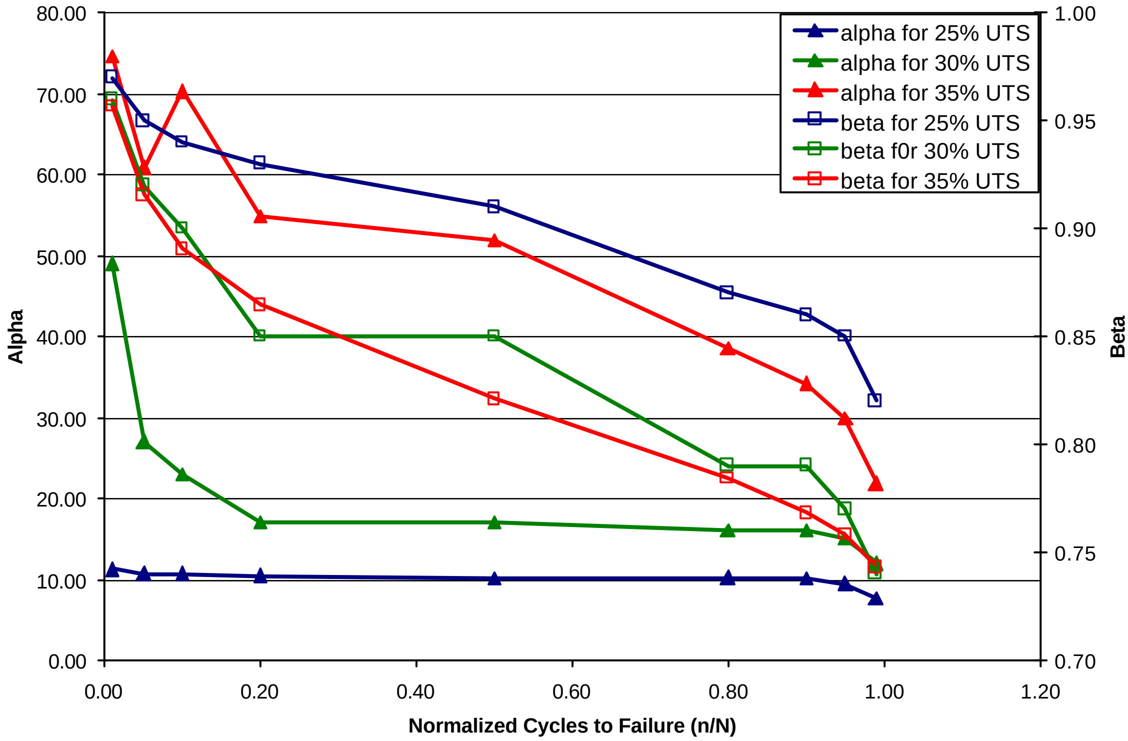

Figure 8: Weibull parameters evolution over normalized time

Model Verification

The validation of the model is a straightforward process. Simply speaking a number of samples were fatigued at the 35% UTS for the prescribed 18750 cycles. After each sample was fatigued it was immediately failed in tension to determine the remaining tensile strength. The Weibull parameters determined experimentally are $\alpha = 10.9$ and $\beta = 23.6$ E 3 PSI or 68% of the original strength. Also 23% of the samples tested failed in fatigue before they reached the 18750 cycle mark. Although the shape parameter was twice (experimentally) that the predicted, the methodology was able to predict the percentage failed and approximate the remaining strength within 2%.

Summary & Remarks

We have presented a perspective on the development of design guides for FRP composites based on reliability and the philosophy presented by the AASHTO LRFD approach. As these guides are presently developed for systems based on as processed conditions we propose a means to include service environment degradation in the determination of resistance reduction factors, ϕ. Provided with this factor, the engineer can properly select material and component size in completing an FRP design suitable for the longevity in a particular service environment.

Preliminary modeling efforts of a simple case show promising results. Currently the simulation is in good agreement with experimental data with regards to the residual strength prediction. The current model doe however not accurately predict the Weibull shape factor. Completing such a simulation requires further work to understand combined environments and their influence on residual strength and stiffness. Further validation tests must be conducted to assess the accuracy of the approach and its limits. Ultimately the approach must be applied to the growing numbers of systems and FRP designs that hold promise for routine use in the highway bridge structures. Once the residual strength distribution can be adequately predicted the final step would be to implement the LRFD algorithms to determine the resistance factors for the systems at future times.

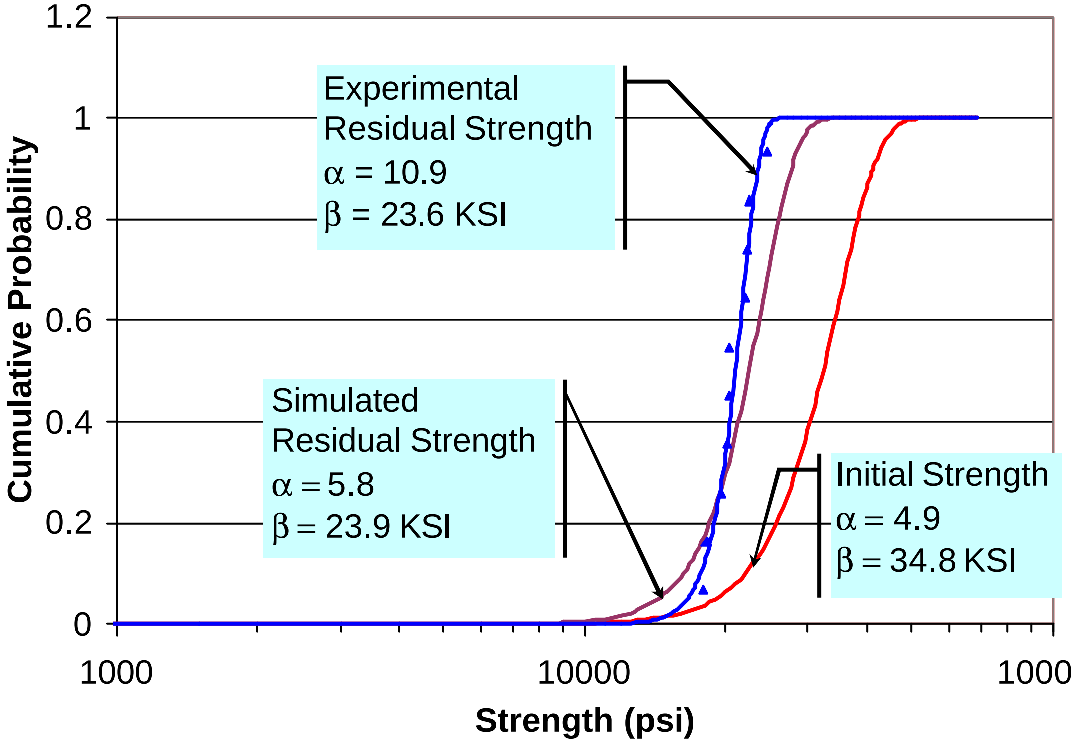

Figure 9: Weibull parameters evolution over normalized time

References

1. AASHTO. 1998. LRFD Bridge Design Specification, 2nd edition. *American Association of State Highway and Transportation Officials*. Washington, D.C.
2. McBagonluri, F., Garcia, K., Hayes, M., Verghese, N. & Lesko, J. J. 2000. Characterization of Fatigue and Combined Environment on Durability Performance of Glass/Vinyl Ester Composite for Infrastructure Applications. *International Journal of Fatigue* 22(1): 53-64.
3. Neely, W. D. 2000. Evaluation of the In-Service Performance of the Tom's Creek Bridge. *M.S. Thesis*. Virginia Tech
4. Phifer, S.P., Verghese, K.N.E., Haramis, J. & Lesko, J.J. 1999. Temperature-Moisture-Mechanical Response of Vinyl Ester Resin and Pultruded Vinyl Ester/E-glass Laminated Composites. *ANTEC 2000, May 7-11*.Orlando:Florida
5. Reifsnider, K.L., Lesko, J. J., & Case, S. W. 1995. Kinetic Methods for Prediction of Damage Tolerance of High Temperature Polymer Composites. *Composites '95: Recent Advances in Japan and the United States, I. Kimpara, H. Miyairi, and N. Takeda, Ed., Proceedings Japan-U.S. CCM-VII*, Kyoto:Japan.49-55.
6. Reifsnider, K.L., Iyengar, N., Case, S.W., & Xu, Y.L. 1996. Damage Tolerance and Durability of Fibrous Material Systems: A Micro-Kinetic Approach. *Durability Analysis of Structural Composite Systems, A. H. Cardon (Ed).* A. A. Rotterdam: Balkema.123-144.
7. Strongwell. 1994. *NIST Advanced Technology Program. Development of Innovative Manufacturing Technology to Produce Large Phenolic Composite Shapes*.
8. Strongwell. 2000. *EXTREN DWB™ Design Guide*.
9. Weibull, & Waloddi. 1951. A Statistical Distribution Function of Wide Applicability. *J. of Applied Mechanics*: 293-297
10. Weibull & Waloddi. 1949. A Statistical Representation of Fatigue Failures in Solids. *Transitions of the Royal Institute of Technology* 27: Stockholm.